

Coordinate Geometry 2:
Coordinate Properties of Transformations

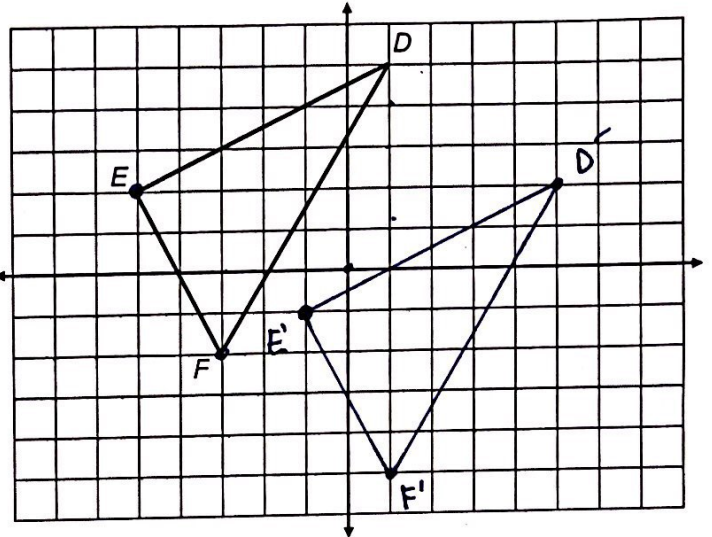
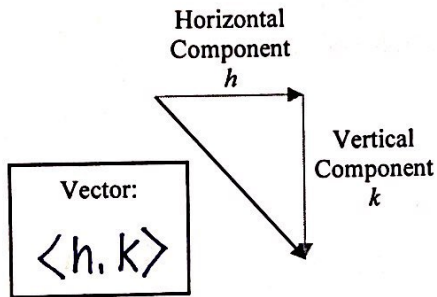
Translations on the Coordinate Plane:

- use an order pair rule (using addition and subtraction)

- example:

$$(x, y) \rightarrow (x + 4, y - 3)$$

Vector Notation:



- example:

translation vector: $\langle 4, -3 \rangle$

$$\begin{aligned} F(-3, -2) &\rightarrow F'(1, -5) \\ E(-5, 2) &\rightarrow E'(-1, -1) \\ D(1, 5) &\rightarrow D'(5, 2) \end{aligned}$$

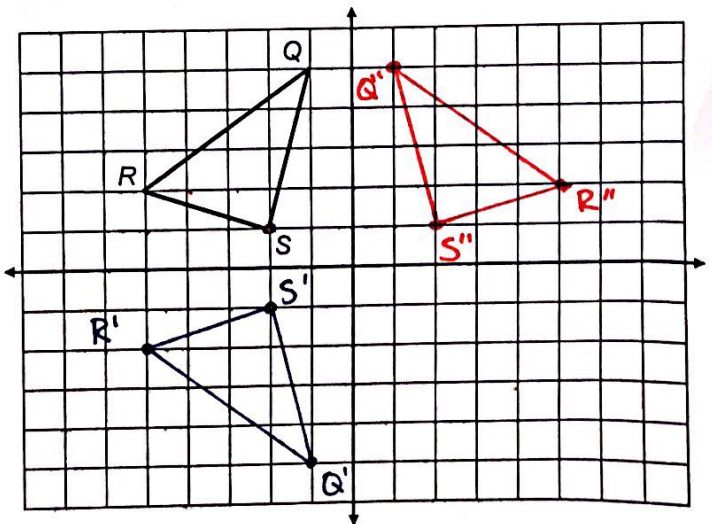
Reflections on the Coordinate Plane:

- using the x -axis as the reflecting line:

$$r_{x\text{-axis}}(x, y) \rightarrow (\underline{x}, \underline{-y})$$

- using the y -axis as the reflecting line:

$$r_{y\text{-axis}}(x, y) \rightarrow (\underline{-x}, \underline{y})$$



Rotations on the Coordinate Plane:

- example:

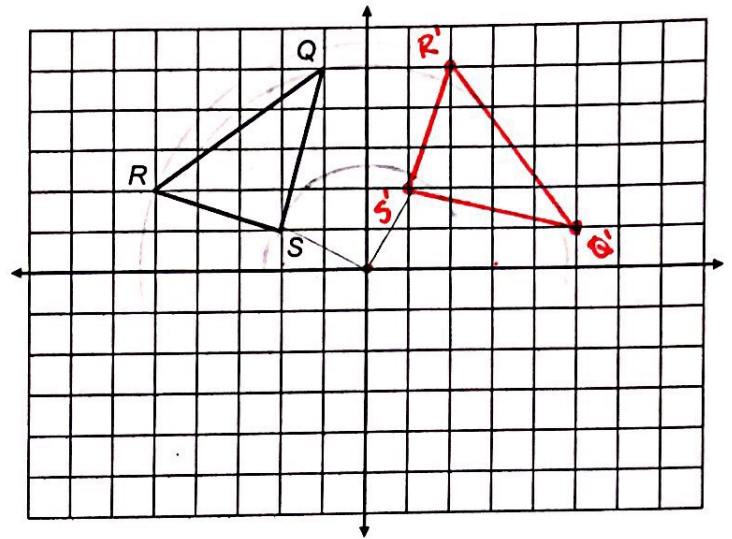
$$(x, y) \rightarrow (y, -x)$$

- Describe the transformation:

$$R (-5, 2) \rightarrow R' (2, 5)$$

$$Q (-1, 5) \rightarrow Q' (5, 1)$$

$$S (-2, 1) \rightarrow S' (1, 2)$$



Use the remaining coordinate planes to determine the properties of the given transformations, and complete the blanks in the following conjecture:

Coordinate Transformations Conjecture:

- The ordered pair rule $(x, y) \rightarrow (-x, y)$ is a reflection across y-axis
- The ordered pair rule $(x, y) \rightarrow (x, -y)$ is a reflection across x-axis
- The ordered pair rule $(x, y) \rightarrow (-x, -y)$ is a rotation^{180°} about the origin
- The ordered pair rule $(x, y) \rightarrow (y, x)$ is a reflection across the line $y=x$
- The ordered pair rule $(x, y) \rightarrow (y, -x)$ is a rotation 90° about the origin clockwise
- The ordered pair rule $(x, y) \rightarrow (-y, x)$ is a rotation 90° about the origin counterclockwise
- The ordered pair rule $(x, y) \rightarrow (-y, -x)$ is a reflection across the line $y=-x$

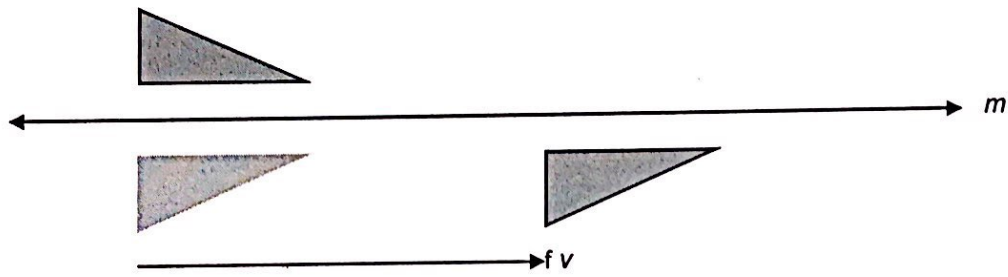
Coordinate Geometry 3:
Compositions of Transformations in the Coordinate Plane

Compositions of Transformations

Definition: A composition of transformations occurs when one isometry is applied to a pre-image, then another isometry is applied to that image to create a final image

Example: **Glide Reflection:** (the fourth and final isometry!)

- the composition of a reflection and a translation



Notation: if r_m is a reflection over line m , and T is a translation, the glide

reflection is given by: $T \cdot r_m$
 second isometry \uparrow \uparrow 1st isometry
 "following"

Equivalent Isometries:

- when a single isometry yields the same final image as
 a composition of other isometries

Example 1:

- Perform the following translation on ΔABC to get $\Delta A'B'C'$:

$$(x, y) \rightarrow (x + 2, y - 6)$$

- Perform the following translation on $\Delta A'B'C'$ to get $\Delta A''B''C''$:

$$(x, y) \rightarrow (x + 3, y + 4)$$

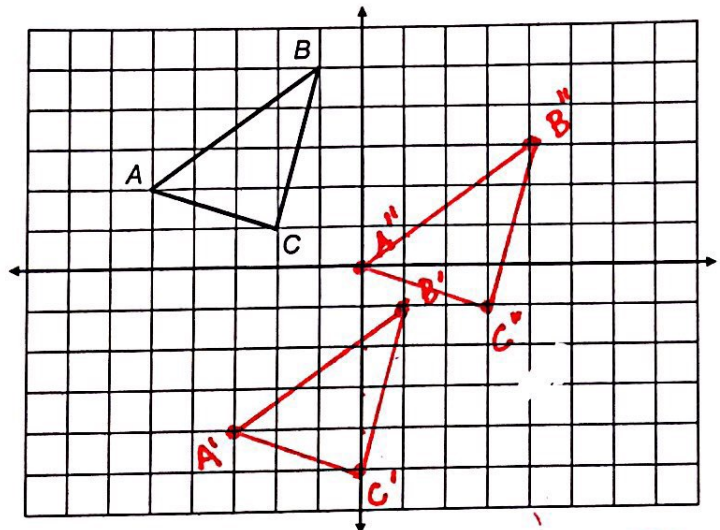
- Is the composite image still an isometry? Which one?

yes, rigid isometry

- Describe the equivalent isometry that moves ΔABC to $\Delta A''B''C''$?

single translation

$$\langle x+5, y-2 \rangle$$



$$A(-5, 2) \rightarrow A'(-3, -4) \rightarrow A''(0, 6)$$

$$B(-1, 5) \rightarrow B'(1, -1) \rightarrow B''(4, +3)$$

$$C(-2, 1) \rightarrow C'(0, -5) \rightarrow C''(3, -1)$$

Example 2:

- Perform the following translation on ΔDEF to get $\Delta D'E'F'$:

$$\Gamma_{x\text{-axis}}(x, y)$$

- Perform the following translation on $\Delta R'Q'S'$ to get $\Delta D''E''F''$:

$$\Gamma_{y\text{-axis}}(x, y)$$

- Is the composite image still an isometry? Which one?

yes, rigid isometry

- Describe the equivalent isometry that moves ΔDEF to $\Delta D''E''F''$?

single rotation \rightarrow toward 1st axis to the 2nd

